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6 Steady State Power System Voltage Stability Analysis and Control with FACTS

Voltage stability analysis and control become increasingly important as the systems are being operated closer to their stability limits including voltage stability limits. This is due to the fact that there is lack of network investments and there are large amounts of power transactions across regions for economical reasons in electricity market environments. It has been recognized that a number of the system blackouts including the recent blackouts that happened in North America and Europe are related to voltage instabilities of the systems.

For voltage stability analysis, a number of special techniques such as power flow based methods and dynamic simulations methods have been proposed and have been used in electric utilities [1]-[4]. Power flow based methods, which are considered as steady state analysis methods, include the standard power flow methods [5], continuation power flow methods [6]-[11], optimization methods [18]-[22], modal methods [2], singular decomposition methods [1], etc.

This chapter focuses on the methods for steady state power system voltage stability analysis and control with FACTS. The objectives of this chapter are summarized as follows:

1. to discuss steady state power system voltage stability analysis using continuation power flow techniques,
2. to formulate steady state power system voltage stability problem as an OPF problem,
3. to investigate FACTS control in steady state power system voltage stability analysis,
4. to discuss the transfer capability calculations using continuation power flow and optimal power flow methods,
5. to discuss security constrained OPF for transfer capability limit determination.

6.1 Continuation Power Flow Methods for Steady State Voltage Stability Analysis

6.1.1 Formulation of Continuation Power Flow

Predictor Step. To simulate load change, Pd_i and Qd_i , may be represented by:

$$Pd_i = Pd_i^0 (1 + \lambda * KPd_i) \quad (6.1)$$

$$Qd_i^p = Qd_i^0 (1 + \lambda * KQd_i) \quad (6.2)$$

where Pd_i^0 and Qd_i^0 are the base case active and reactive load powers of phase p at bus i . λ is the loading factor, which characterize the change of the load. The ratio of KPd_i^p / KQd_i^p is constant to maintain a constant power factor.

Similarly, to simulate generation change, Pg_i and Qg_i , are represented as functions of λ and given by:

$$Pg_i = Pg_i^0 (1 + \lambda * KPg_i) \quad (6.3)$$

$$Qg_i = Qg_i^0 (1 + \lambda * KQg_i) \quad (6.4)$$

where Pg_i^0 and Qg_i^0 are the total active and reactive powers of the generator of the base case. The ratio of KPg_i / KQg_i is constant to maintain constant power factor for a PQ machine. For a PV machine, equation (6.4) is not required. For a PQ machine, when the reactive limit is violated, Qg_i should be kept at the limit and equation (6.4) is also not required.

The nonlinear power flow equations are augmented by an extra variable λ as follows:

$$f(x, \lambda) = 0 \quad (6.5)$$

where $f(x, \lambda)$ represents the whole set of power flow mismatch equations.

The predictor step is used to provide an approximate point of the next solution. A prediction of the next solution is made by taking an appropriately sized step in the direction tangent to the solution path.

To solve (6.5), the continuation algorithm with predictor and corrector steps can be used. Linearizing (6.5), we have:

$$df(x, \lambda) = f_x dx + f_\lambda d\lambda = 0 \quad (6.6)$$

In order to solve (6.6), one more equation is needed. If we choose a non-zero magnitude for one of the tangent vector and keep its change as ± 1 , one extra equation can be obtained:

$$t_k = \pm 1 \quad (6.7)$$

where t_k is a non-zero element of the tangent vector dx .

Combining (6.6) and (6.7), we can get a set of equations where the tangent vector dx and $d\lambda$ are unknown variables:

$$\begin{bmatrix} \frac{f_x}{e_k} & \frac{f_\lambda}{e_k} \end{bmatrix} \begin{bmatrix} dx \\ d\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ \pm 1 \end{bmatrix} \quad (6.8)$$

where e_k is a row vector with all elements zero except for K^{th} , which equals one. In (6.8), whether +1 or -1 is used depends on how the K^{th} state variable is changing as the solution is being traced. After solving (6.8), the prediction of the next solution may be given by:

$$\begin{bmatrix} x^* \\ \lambda^* \end{bmatrix} = \begin{bmatrix} x \\ \lambda \end{bmatrix} + \sigma \begin{bmatrix} dx \\ d\lambda \end{bmatrix} \quad (6.9)$$

where * denotes the estimated solution of the next step while σ is a scalar, which represents the step size.

Corrector Step. The corrector step is to solve the augmented Newton power flow equation with the predicted solution in (6.9) as the initial point. In the augmented Newton power flow algorithm an extra equation is included and λ is taken as a variable. The augmented Newton power flow equation may be given by:

$$\begin{bmatrix} f(x, \lambda) \\ x_k - \eta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (6.10)$$

where η , which is determined by (6.10), is the predicted value of the continuation parameter x_k . The determination of the continuation parameter is shown in the following solution procedure.

The corrector equation (6.10), which consists of a set of augmented nonlinear equations, can be solved iteratively by Newton's approach as follows:

$$\begin{bmatrix} \frac{f_x}{e_k} & \frac{f_\lambda}{e_k} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta \lambda \end{bmatrix} = - \begin{bmatrix} f(x, \lambda) \\ x_k - \eta \end{bmatrix} \quad (6.11)$$

6.1.2 Modeling of Operating Limits of Synchronous Machines

Normally a generator terminal bus is considered as a PV bus, at which the voltage magnitude is specified while the rotor, stator currents and reactive power limits are being monitored according to the capability curve of the generator. The operating limits of a generator that should be satisfied are as follows:

$$I_a \leq I_a^{\max} \quad (6.12)$$

$$I_f^{\min} \leq I_f \leq I_f^{\max} \text{ or } E_f^{\min} \leq E_f \leq E_f^{\max} \quad (6.13)$$

$$Pg^{\min} \leq Pg \leq Pg^{\max} \quad (6.14)$$

$$Qg^{\min}(Pg) \leq Qg \leq Qg^{\max}(Pg) \quad (6.15)$$

where $I_a^{\max}$ is the current limit of the generator stator winding, $I_f^{\max}$ and $I_f^{\min}$ are the maximum and minimum current limits of the generator rotor winding, respectively, while $E_f^{\max}$ and $E_f^{\min}$ are the corresponding excitation voltage limits.

$Pg^{\max}$ and $Pg^{\min}$ are the maximum and minimum reactive power limits determined by the capability curve, which are used in continuation power flow analysis. $Qg^{\max}$ and $Qg^{\min}$ are the maximum and minimum reactive power limits determined by the capability curve, which are usually the functions of active power generation.

When one of the inequalities above is violated, the variable is kept at the limit while the voltage control constraint is released. However, when more than one inequality is violated, the technique proposed in [12] can be applied to identify the dominant constraint, and then the dominant constraint is enforced while the other constraints are monitored.

6.1.3 Solution Procedure of Continuation Power Flow

The general solution procedure for the Continuation Three-Phase Power Flow is given as follows:

Step 0: Run three-phase power flow when Pd_i , Qd_i , Pg_i and Qg_i are set to Pd_i^0 , Qd_i^0 , Pg_i^0 and Qg_i^0 , respectively. The initial point for tracing the PV curves is found.

Step 1: Predictor Step

- (a) Solve (6.8) and get the tangent vector $[dx, d\lambda]^t$;
- (b) Use (6.9) to find the predicted solution of the next step.
- (c) Choose the continuation parameter by evaluating $x_k : t_k = \max(|dx_i|)$.
- (d) Check whether the critical point (maximum loading point) has been passed by evaluating the sign of $d\lambda$. If $d\lambda$ changes its sign from positive to negative, then the critical point has just passed.
- (e) Check whether $\lambda^* < 0$ (Note $0 \leq \lambda \leq \lambda_{\max}$). If this is true, go to Step 3.

Step 2: Corrector Step

- (a) According to the chosen continuation parameter to form the aug-

- mented equation (6.10);
- (b) Form and solve the Newton equation (6.11);
- (c) Update the Newton solution and continue the iterations until the corrector step converges to a solution with a given tolerance;
- (d) Go to Step 1.

Step 3: Output solutions of the PV curves.

For tracing the upper portion of PV curves, λ may be taken as the continuation parameter. If, at the predictor step, $d\lambda$ is changed from positive to negative, then the critical point has just passed, and the continuation parameter may be changed from loading factor λ to bus voltage magnitude. The bus voltage magnitude with the largest decrease may be chosen as the continuation parameter.

The negative voltage sensitivities, at or near the critical point, with respect to the loading factor λ are very useful information in identifying the vulnerable system buses. The bigger the voltage sensitivities, the more vulnerable the system buses are.

The continuation power flow described above can be applied to two situations. The first situation is in the determination of system loadability limit while the second is in the determination of system transfer capability limit. If, in the analysis, voltage limits of load buses and thermal limits of transmission lines are not considered, the system loadability limit or the transfer capability limit is, in principle, corresponding to the system voltage stability limit. However, if voltage limits of load buses and thermal limits of transmission lines are considered, in principle the system loadability limit or the transfer capability limit may be lower than the corresponding system voltage stability limit.

6.1.4 Modeling of FACTS-Control in Continuation Power Flow

In principle, similar to the power flow analysis, the models for FACTS-devices such as SVC, TCSC, STATCOM, SSSC, UPFC, IPFC, GUPFC and VSC HVDC are applicable to the continuation power flow for the steady state voltage stability analysis. In addition to the FACTS-devices, other control devices such as explicit model of excitation systems, tap changer control may be considered.

6.1.5 Numerical Results

In the following, numerical results are carried out on the IEEE 30-bus system and the IEEE 118-bus system. The single-line diagram of the IEEE 30-bus system is shown in Fig. 2.2, while the single-line diagram of the IEEE 118-bus system is presented in Fig. 6.1.

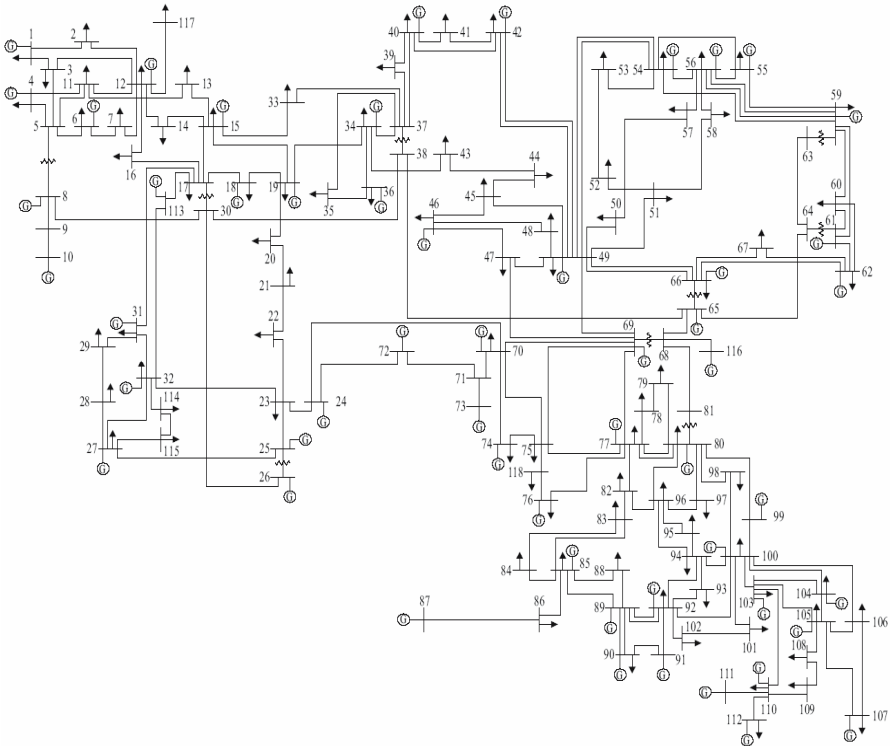


Fig. 6.1. IEEE 118-bus system

6.1.5.1 System Loadability with FACTS-Devices

Two cases for the IEEE 30-bus system and the IEEE 118-bus system have been studied. The maximum loading factors of these two cases are shown in Table 6.1.

For the IEEE 30-bus system, the candidate buses, at which STATCOMs are installed, are the buses with larger voltage sensitivities with respect to system loading factor λ at the voltage collapse point or the nose point. It is found that three largest voltage sensitivities are at buses 28, 29 and 30. A STATCOM is installed at bus 29 of the IEEE 30-bus system. The maximum loading factor is given by Table 6.2, which shows an increase of the maximum voltage stability limit by 33%.

Table 6.1. Maximum loading factors

Case No.	System	Maximum loading factor $\lambda_{\max}$
Case 1	IEEE 30-bus system	2.08
Case 2	IEEE 118-bus system	2.25

Table 6.2. Maximum loading factors

Case No.	System	Maximum loading factor $\lambda_{\max}$	Increase of the maximum loading in percentage using FACTS control
Case 3	IEEE 30-bus system	2.77	33%
Case 4	IEEE 118-bus system	2.66	18%

For the IEEE 118-bus system without STATCOM, it is found that the largest voltage sensitivities at the voltage collapse point or the nose point are at buses 38, 43, 44, 45. Four STATCOM are installed at these buses, respectively. The maximum loading factor for the IEEE 118-bus system with 4 STATCOMs is presented in Table 6.1, which shows an increase of the voltage stability limit by 18%.

It has been found that for case 3 and case 4, shunt reactive power control using STATCOM (or SVC) is very effective while series reactive power compensation control using SSSC and series-shunt reactive power compensation using UPFC are not effective.

6.1.5.2 Effect of Load Models

Without considering the frequency effect, a general static load model may be given by:

$$Pd_i^0 = Pd_i^{norm} (a_{i0} + a_{i1}V_i + a_{i2}V_i^2) \quad (6.16)$$

$$Qd_i^0 = Qd_i^{norm} (b_{i0} + b_{i1}V_i + b_{i2}V_i^2) \quad (6.17)$$

where subscript i denotes the bus number. Pd_i^{norm} and Qd_i^{norm} are the active and reactive powers at nominal voltage. a_{i0} and b_{i0} represent the constant power components; a_{i1} and b_{i1} represent the constant current components; a_{i2} and b_{i2} represent the constant impedance components. The model in (6.16) and (6.17) is also known as ZIP model where Z represents impedance, I represents current, and P represents power. The parameters in (6.16) and (6.17) should satisfy the following equations:

$$a_{i0} + a_{i1} + a_{i2} = 1 \quad (6.18)$$

$$b_{i0} + b_{i1} + b_{i2} = 1 \quad (6.19)$$

In order to investigate the effects of different load models on voltage stability limits, cases 5-10 for the IEEE 30-bus system, which are presented in Table 6.3. The PV curves of bus 27 are shown in Fig. 6.2-Fig. 6.7, respectively.

Table 6.3. Case studies with different load models for the IEEE 30-bus system

Case No.	Load model parameters
Case 5	$a_{i0} = b_{i0} = 1.0$, $a_{i1} = b_{i1} = 0.0$, $a_{i2} = b_{i2} = 0.0$, PQ load
Case 6	$a_{i0} = b_{i0} = 0.6$, $a_{i1} = b_{i1} = 0.4$, $a_{i2} = b_{i2} = 0.0$
Case 7	$a_{i0} = b_{i0} = 0.6$, $a_{i1} = b_{i1} = 0.0$, $a_{i2} = b_{i2} = 0.4$
Case 8	$a_{i0} = b_{i0} = 0.6$, $a_{i1} = b_{i1} = 0.2$, $a_{i2} = b_{i2} = 0.2$
Case 9	$a_{i0} = b_{i0} = 0.0$, $a_{i1} = b_{i1} = 1.0$, $a_{i2} = b_{i2} = 0.0$, Current load
Case 10	$a_{i0} = b_{i0} = 0.0$, $a_{i1} = b_{i1} = 0.0$, $a_{i2} = b_{i2} = 1.0$, Impedance load

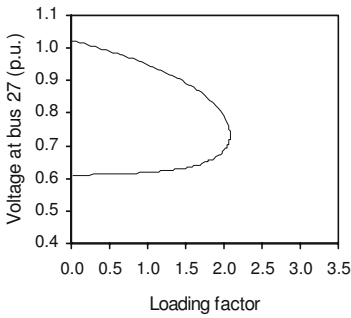


Fig. 6.2. PV curve at bus 27 for case 5

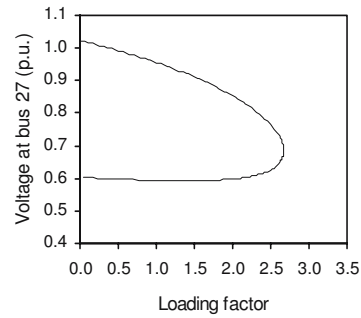


Fig. 6.3. PV curve at bus 27 for case 6

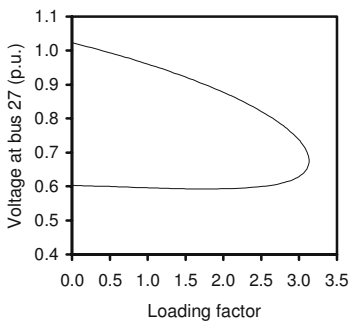


Fig. 6.4. PV curve at bus 27 for case 7

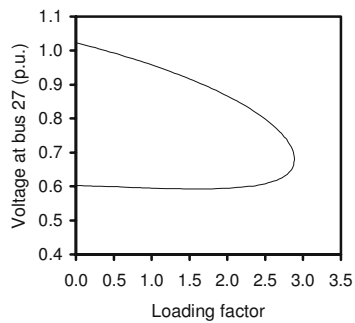


Fig. 6.5. PV curve at bus 27 for case 8

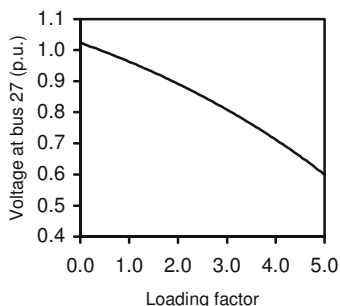


Fig. 6.6. PV curve at bus 27 for case 9

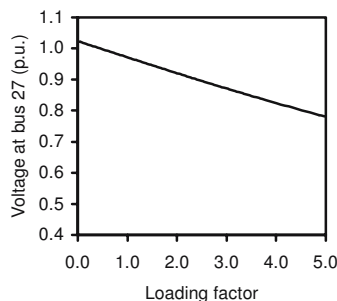


Fig. 6.7. PV curve at bus 27 for case 10

From Figures 6.2 to 6.5 and Table 6.3, it can be seen that for the constant power model, the maximum loading factor is the minimal. It should be pointed out, that for case 9 and case 10, there is no nose point available. From these examples, it is clear that load models play a very important role in voltage stability analysis.

6.1.5.3 System Transfer Capability with FACTS-Devices

Two cases for the IEEE 30-bus system without and with FACTS-devices have been studied. In the study, the IEEE 30 bus system was divided into two areas. The two areas are interconnected by intertie lines: 4-12, 6-9, 6-10, and 28-27 while buses 4, 6, 28 belong to the area 1. The power transfer from the area 1 to the area 2 has been investigated.

The maximum loading factors of these two cases are shown in Table 6.4. The system transfer capability limit here is limited by voltage stability limit while the load bus voltage limits and thermal limits of the transmission lines are not considered. As it has been discussed, the candidate buses, at which STATCOMs are installed, are the buses with larger voltage sensitivities with respect to system loading factor λ at the voltage collapse point or the nose point. It is found that three largest voltage sensitivities are at buses 27, 29 and 30. A STATCOM is installed at bus 29 of the IEEE 30-bus system. The maximum loading factor is given by Table 6.4, which shows an increase of the maximum voltage stability limit by 35%.

Table 6.4. Maximum loading factors

Case No.	System	Maximum loading factor $\lambda_{\max}$
Case 11	IEEE 30-bus system without FACTS	2.15
Case 12	IEEE 30-bus system with FACTS	2.90

It has been found that for case 11 and case 12, shunt reactive power control using STATCOM (or SVC) is very effective while series reactive power compensation control using SSSC and series-shunt reactive power compensation using UPFC are not effective. The reason is that the effectiveness of series FACTS control relies on its global optimal setting.

The advantage of the continuation method is that operating limits such as thermal, voltage and voltage stability limits can be fully taken into account. However, the disadvantages of the method include:

- Adjustment of generation, transformer tap positions, FACTS-controls, etc. in loadability or transfer capability calculations would be very difficult.
- It is heuristic in nature when voltage and thermal limits are considered in loadability or transfer capability calculations.
- The difficulty of implementation of global coordination. For instance, it is difficult to find optimal settings for series FACTS-devices and coordinate their controls.

In nature, continuation power flow belongs to power flow analysis. In principle, techniques that have been successfully applied to solve power flow problems should be applicable to continuation power flow calculations.

6.2 Optimization Methods for Steady State Voltage Stability Analysis

It has been well recognized that optimization methods can be applied to determine the system loadability and transfer capability. However, the definition and formulation of these in literature are not consistent since there are a few possible different formulations of these problems considering different combination of equipment, voltage and thermal constraints. In the following, different formulations for system loadability and transfer capability problems are discussed at first, then numerical examples are given.

6.2.1 Optimization Method for Voltage Stability Limit Determination

The maximum voltage stability limit can be formulated as a nonlinear optimization problem. The objective of the problem is to determine the maximum voltage stability limit for a power system considering either the increase of total system load for the case of loadability determination, or the increase of load at a specified region or buses for the case of transfer capability determination while satisfying generator bus voltage constraints and equipment constraints. The optimization problem may be formulated as follows:

$$\text{Maximize: } \lambda \tag{6.20}$$

subject to:

$$\mathbf{g}(\mathbf{x}, \mathbf{u}, \lambda) = 0 \quad (6.21)$$

$$\mathbf{h}_{\min} \leq \mathbf{h}(\mathbf{x}, \mathbf{u}) \leq \mathbf{h}_{\max} \quad (6.22)$$

where

- $\mathbf{u}$ - the set of control variables
- $\mathbf{x}$ - the set of dependent variables
- $\mathbf{g}(\mathbf{x}, \mathbf{u})$ - the power flow equations, and control equality constraints for FACTS, transformers, generators, etc
- $\mathbf{h}(\mathbf{x}, \mathbf{u})$ - the limits of the control variables, operating limits of power system components such as generators, transformers and FACTS-devices, and voltage constraints at load buses

The problem in (6.20)-(6.22) can be solved by nonlinear interior point methods [19]-[21]. In the problem in (6.20)-(6.22), the bus load may be represented by:

$$P_d = \lambda P_d^0 \quad (6.23)$$

$$Q_d = \lambda Q_d^0 \quad (6.24)$$

where P_d^0 and Q_d^0 are the base case bus active and reactive load powers, and it is assumed that a constant power factor is maintained. It should be pointed out that λ defined here is different from λ used in the continuation power flow analysis as shown in (6.1)-(6.4). In other words, λ defined in (6.23) and (6.24) is corresponding to $\lambda + 1$ in the continuation power flow analysis.

In (6.22), thermal limits of transmission line and voltage constraints at load buses are not included. The maximum voltage stability limit problem in (6.20)-(6.22) is very similar to the continuation power flow problem in section 6.1. The significant difference between the two methods is that the former can be only used to determine the voltage stability limit, while the latter is able to trace the bus PV curves, simulate control sequences and actions, and obtain sensitivity information along the PV curves. The advantages of the former are:

- Coordinated adjustment of control settings of generators, transformers and FACTS-devices, etc.
- Direct consideration of equipment limits and operating limits in the formulation.

6.2.2 Optimization Method for Voltage Security Limit Determination

The maximum loadability or transfer capability limit determination can be formulated as a nonlinear optimization problem. The objective of the problem is to determine the maximum system load increase for a power system considering either the increase of total system load for the case of loadability determination, or the increase of load at a specified region or buses for the case of transfer capability

determination while satisfying bus voltage constraints and equipment constraints. The optimization problem may be formulated, which is very similar to problem in (6.20)-(6.22) except that now voltage constraints at load buses are also considered. The optimization problem can be solved by nonlinear interior point methods [19]-[21].

6.2.3 Optimization Method for Operating Security Limit Determination

The maximum loadability or transfer capability limit determination considering operating security constraints can be formulated as a nonlinear optimization problem. The objective of the problem is to determine the maximum system load increase for a power system considering either the increase of total system load for the case of loadability determination, or the increase of load at a specified region or buses for the case of transfer capability determination while satisfying all bus voltage constraints, thermal constraints of transmission lines, and equipment constraints. The optimization problem may be formulated as follows:

$$\text{Maximize: } \lambda \quad (6.25)$$

subject to:

$$\mathbf{g}(\mathbf{x}, \mathbf{u}, \lambda) = 0 \quad (6.26)$$

$$\mathbf{h}_{\min} \leq \mathbf{h}(\mathbf{x}, \mathbf{u}) \leq \mathbf{h}_{\max} \quad (6.27)$$

where

- $\mathbf{u}$ - the set of control variables
- $\mathbf{x}$ - the set of dependent variables
- $\mathbf{g}(\mathbf{x}, \mathbf{u})$ - the power flow equations, and control equality constraints for FACTS, transformers, generators, etc
- $\mathbf{h}(\mathbf{x}, \mathbf{u})$ - the limits of the control variables, operating limits of power system components such as generators, transformers and FACTS-devices, voltage constraints at all buses, and thermal limits of transmission lines

The optimization problem in (6.25)-(6.27) can be solved by nonlinear interior point methods [19]-[21]. In (6.27), thermal limits of transmission lines and voltage constraints at all buses are included. In transfer capability calculations, when contingencies should be considered, a security-constrained transfer capability problem can be formulated, which will be discussed in section 6.3.

6.2.4 Optimization Method for Power Flow Unsolvability

As the requirements for satisfactory system operation, the region of feasible solutions, satisfying all constraints simultaneously, may not be able to converge. In other words, the power flow or optimal power flow problem is unsolvable. In this situation, the critical question is how to take control actions to restore the solvabil-

ity of the power flow or optimal power problem. In the following, a robust nonlinear OPF formulation which introduces reactive slack variables and load shedding variables in the unsolvable problem is proposed to handle the infeasibility of a solution. It is formulated as:

$$\begin{aligned} \text{Minimize: } & \sum_i^N CQr0_i + CQr1_i * Qr_i + CQr2_i * Qr_i^2 \\ & + \sum_i^N CQc0_i + CQc1_i * Qc_i + CQc2_i * Qc_i^2 \\ & + \sum_i^N CPd0_i + CPd1_i * \Delta Pd_i + CPd2_i * \Delta Pd_i^2 \end{aligned} \quad (6.28)$$

subject to the following constraints:

$$\begin{aligned} Pg_i - Pd_i + \Delta Pd_i - P_i(V, \theta, T) = DP_i(\mathbf{x}) = 0 \\ (i=1, 2, 3, N) \end{aligned} \quad (6.29)$$

$$\begin{aligned} Qg_i - Qd_i + a_i * \Delta Pd_i + Qc_i - Qr_i - Q_i(V, \theta, T) = DQ_i(\mathbf{x}) = 0 \\ (i=1, 2, 3, N) \end{aligned} \quad (6.30)$$

$$\begin{aligned} h_j^{\min} \leq h_j(x) \leq h_j^{\max} \\ (j = 1, 2, \dots, Nh) \end{aligned} \quad (6.31)$$

where

$$\mathbf{x} = [V, \theta, T, Pg, Qg, Qr, Qc, \Delta Pd]^T$$

$\Delta Pd_i, \Delta Qd_i$ - bus active and reactive load shedding, respectively

DP_i, DQ_i - bus active and reactive power mismatch, respectively

$CPd0_i, CPd1_i, CPd2_i$ - bus load shedding cost coefficient

a_i - constant ratio

Qr_i, Qc_i - bus fictitious inductive and capacitive VAR injections, respectively

$CQr0_i, CQr1_i, CQr2_i$ - bus cost coefficients for fictitious inductive VAR injections

$CQc0_i, CQc1_i, CQc2_i$ - bus cost coefficients for fictitious capacitive VAR injections

$h_j^{\min}, h_j^{\max}$ - lower and upper limits of inequality

The main idea of the optimization problem for restoring unsolvability is to minimize the cost of control actions in (6.29) while satisfying voltage and thermal constraints and determining the optimal values of reactive power and load shedding controls. If the resulting fictitious inductive and capacitive VAR injections cost coefficients are set to very high values, the solvability of the power flow problem is restored by load shedding only. However, if the load shedding cost coefficients are

set to very high values, the solvability is restored by reactive power compensation. For some unsolvable situations, the power flow solution may be restored by combination of reactive power and load shedding controls. The optimal solution of the problem indicates the minimum cost of the control actions should be taken to make the power flow problem solvable.

6.2.5 Numerical Examples

Test cases are carried out on the IEEE 30-bus system and IEEE 118-bus system. For all cases tested, the convergence criteria are:

1. Complementary gap $C_{gap} \leq 5.0e^{-4}$
2. Barrier parameter $\mu \leq 1.0e^{-4}$
3. Maximum mismatch of the Newton equation $\| \mathbf{b} \|_{\infty} \leq 1.0e^{-4}$ p.u.

6.2.5.1 IEEE 30-Bus System Results

In the study, the IEEE 30-bus system was divided into two areas. The two areas are interconnected by tie-lines: 4-12, 6-9, 6-10, and 28-27 while buses 4, 6, 28 belong to the area 1. The transfer from the area 1 to the area 2 has been carried out. The two cases are presented as follows:

Case 1: This is a case for transfer capability computation without FACTS-devices.

Case 2: This is similar to the case 1 except that there is a SSSC installed on line 2-1.

For case 1, the transfer capabilities considering the voltage stability limit, voltage security limit and operating security limit, respectively, as discussed in previous sections, are shown in Fig.6.8. The corresponding models are referred to model 1 for voltage stability limit, model 2 for voltage security limit and model 3 for operating security limit. In Fig. 6.8, vertical axis shows the transfer capability (TC), which is described by λ . Comparing the transfer capabilities shown in Fig. 6.8, it can be seen that the voltage stability limit is bigger than the voltage security limit and operating security limit, and the voltage security limit is bigger than the operating security limit.

The transfer capabilities considering the operating security limits for case 1 and 2 are shown in Fig. 6.9. For case 2, the transfer capability has been increased by around 5%.

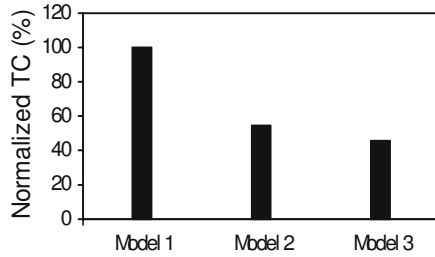


Fig. 6.8. The system transfer capabilities of case 1 using different models

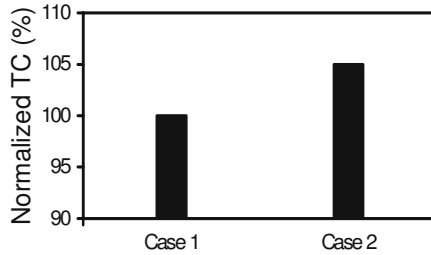


Fig. 6.9. The system transfer capabilities for case 1 and case 2

6.2.5.2 IEEE 118-Bus System Results

In the study on the IEEE 118-bus system, it is assumed that the whole system includes two areas, which are interconnected by tie-lines: 15-33, 19-34, 30-38, and 23-24 while buses 15, 19, 30, 23 belong to the area 1. The transfer capability from the area 1 to the area 2 has been carried out. Three cases are presented as follows:

Case 3: This is a case of the IEEE 118-bus system without FACTS-devices.

Case 4: This is similar to the Case 3 except that there is a GUPFC installed at bus 30 and on lines 30-38, 30-8.

Case 5: This is similar to the Case 4 except that there is a second UPFC further installed at bus 25 and on line 25-27.

The normalized transfer capabilities for the cases 3-5 are shown in Fig. 6.10. It can be seen from this figure that the transfer capabilities can be increased using FACTS-devices.

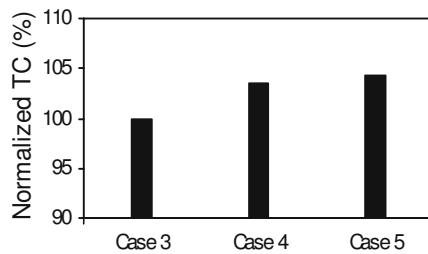


Fig. 6.10. The system transfer capabilities for cases 3-5

6.3 Security Constrained Optimal Power Flow for Transfer Capability Calculations

It has been well recognized that in the operation of electric power markets, determination of the transfer capabilities of the transmission system is a very important analysis function. Transfer capability of electric power systems is limited by a number of different mechanisms, including thermal, voltage and stability constraints, which is characterized by the so-called Available Transfer Capability (ATC) [13][14]. The comprehensive definition of the transfer capability is referred to [14].

The Transfer Capability (TC) computation methods in literature can be classified into, (a) DC power flow calculation method (or linear method) (b) repeated power flow calculation method (c) continuation power flow method (d) OPF and security-constrained OPF methods. The DC power flow calculation method has been implemented in a commercial software product called MUST [15]. The advantage of such method is its simplicity in terms of formulation and computation. In the method, voltage and voltage stability limits are not considered. It has been recognized that neglecting the reactive power and voltage influence in TC may generate errors that in certain conditions could drive the computation to be wrong or at least give inaccurate results. The repeated power flow calculation method has been proposed [16]. The repeated power flow calculation method is heuristic in nature. The computational effort of the method is significantly higher compared with other methods. The continuation method for TC has been reported and implemented in commercial software products [17]. The advantage of the continuation method is that various operating limits such as thermal, voltage and voltage stability limits can be fully taken into account. The disadvantage of this method is that adjustment of generation, transformer tap positions, FACTS controls, etc in TC calculations would be very difficult if not impossible. Similar to the DC power flow calculation method and repeated power flow calculation method, continuation power flow method could not be used in an integrated contingency-constrained analysis framework. In such a situation, sequential heuristic TC calcu-

lations are used instead. Solution of TC by a successive linear programming based OPF has been proposed [18]. Non-linear interior point OPF algorithms for TC calculations have also been proposed [19]-[21]. However, most OPF based TC methods are based on single state calculations. It is recognized that the single state optimization based approaches have difficulty to deal with control actions such as preventive and/or corrective controls. The deficiencies of the current TC computational methods are:

- lack of couplings between base case and contingencies,
- lack of adequate consideration of reactive power/voltage effects and/or voltage stability effects,
- lack of modeling of FACTS-devices in transfer capability determination.

In order to handle the deficiencies of the current TC computation methods, in [22] the TC computation problem has been formulated as a general contingency-constrained optimization problem and has been solved by the nonlinear interior point optimization algorithms. The TC computational method proposed has the following features:

- Considering various operating limits and contingency constraints.
- Incorporating corrective or/and preventive control actions in the united framework.
- Modeling of FACTS-devices.
- Solving simultaneously the base case and contingencies in a united optimization framework.

6.3.1 Unified Transfer Capability Computation Method with Security Constraints

A unified transfer capability computation problem with security constraints may be formulated as:

Objective function:

$$\max f(\mathbf{y}) = \lambda \text{ or } \min -f(\mathbf{y}) \quad (6.32)$$

subject to the following constraints:

Base case constraints:

$$\mathbf{g}_0(\mathbf{y}_0) = \mathbf{0} \quad (6.33)$$

$$\mathbf{h}_0^{\min} \leq \mathbf{h}_0(\mathbf{y}_0) \leq \mathbf{h}_0^{\max} \quad (6.34)$$

Contingency constraints:

$$\mathbf{g}_i(\mathbf{y}_i) = \mathbf{0} \quad (i = 1, 2, \dots, Nc) \quad (6.35)$$

$$\mathbf{h}_i^{\min} \leq \mathbf{h}_i(\mathbf{y}_0) \leq \mathbf{h}_i^{\max} \quad (i = 1, 2, \dots, Nc) \quad (6.36)$$

where subscripts 0 and i indicate base case and contingencies, respectively. N_c is the total number of contingencies. $\mathbf{g}_0(\mathbf{y}_0)$ and $\mathbf{h}_0(\mathbf{y}_0)$ are base case equalities and inequalities, respectively. While $\mathbf{g}_i(\mathbf{y}_i)$ and $\mathbf{h}_i(\mathbf{y}_i)$ are equalities and inequalities respectively for contingency i . $\mathbf{y} = [\mathbf{x}, \mathbf{u}, \lambda]^T$ is the system variable vector. $\mathbf{u}$ is the control variable vector with preventive control actions. λ is a scalar parameter, which represents the loading factor. Modeling of FACTS-devices in power system network analysis can be found in [23]-[25].

Without loss of generality, preventive control actions may be formulated as:

Preventive control actions:

$$\mathbf{u}_0 = \mathbf{u}_i \quad (i=1,2, \dots, N_c) \quad (6.37)$$

where $\mathbf{u}_0$, $\mathbf{u}_i$ are base case and contingency control vectors respectively.

The problem in (6.32)-(6.37) is a unified security constrained transfer capability computation problem. In this problem, bus load may be represented by:

$$P_d = \lambda P_d^0 \quad (6.38)$$

$$Q_d = \lambda Q_d^0 \quad (6.39)$$

where P_d^0 and Q_d^0 are base case bus active and reactive load powers, and it is assumed that a constant power factor is maintained.

6.3.2 Solution of Unified Security Constrained Transfer Capability Problem by Nonlinear Interior Point Method

Mathematically, the unified transfer capability computation problem is an optimization problem, which may be solved by nonlinear interior point methods. The nonlinear OPF problem given in (6.32)-(6.37) can be solved by the nonlinear interior point methods [26][27], which include three important achievements in optimization. Those achievements are Fiacco & McCormick's barrier method for optimization with inequalities, Lagrange's method for optimization with equalities and Newton's method for solving nonlinear equations [26]-[28].

By applying Fiacco & McCormick's barrier method, the unified OPF problem (6.32)-(6.37) can be transformed into the following equivalent OPF problem:

Objective:

$$\min \left\{ \begin{aligned} & -f(\mathbf{y}) - \mu \sum_{j=1}^{N_h} \ln(sl_{0j}) - \mu \sum_{j=1}^{N_h} \ln(su_{0j}) \\ & - \mu \sum_{i=1}^{N_c} \sum_{j=1}^{N_h} \ln(sl_{ij}) - \mu \sum_{i=1}^{N_c} \sum_{j=1}^{N_h} \ln(su_{ij}) \end{aligned} \right\} \quad (6.40)$$

subject to the following equality constraints:

$$\mathbf{g}_0(\mathbf{y}_0) = \mathbf{0} \quad (6.41)$$

$$\mathbf{h}_0(\mathbf{y}_0) - \mathbf{sl}_0 - \mathbf{h}_0^{\min} = \mathbf{0} \quad (6.42)$$

$$\mathbf{h}_0(\mathbf{y}_0) + \mathbf{su}_0 - \mathbf{h}_0^{\max} = \mathbf{0} \quad (6.43)$$

$$\mathbf{g}_i(\mathbf{y}_i) = \mathbf{0} \quad (i = 1, 2, \dots, Nc) \quad (6.44)$$

$$\mathbf{h}_i(\mathbf{y}_i) - \mathbf{sl}_i - \mathbf{h}_i^{\min} = \mathbf{0} \quad (i = 1, 2, \dots, Nc) \quad (6.45)$$

$$\mathbf{h}_i(\mathbf{y}_i) + \mathbf{su}_i - \mathbf{h}_i^{\max} = \mathbf{0} \quad (i = 1, 2, \dots, Nc) \quad (6.46)$$

$$\mathbf{u}_0 - \mathbf{u}_i = \mathbf{0} \quad (i = 1, 2, \dots, Nc) \quad (6.47)$$

where $\mu > 0$, $\mathbf{sl}_0 > \mathbf{0}$, $\mathbf{su}_0 > \mathbf{0}$, $\mathbf{sl}_i > \mathbf{0}$ and $\mathbf{su}_i > \mathbf{0}$. Nh is the number of double sided inequalities.

The Lagrangian function for equalities optimization of problem (6.40)-(6.47) is:

$$\begin{aligned} L = & -f(\mathbf{y}) - \mu \sum_{j=1}^{Nh} \ln(sl_{0j}) - \mu \sum_{j=1}^{Nh} \ln(su_{0j}) \\ & - \mu \sum_{i=1}^{Nc} \sum_{j=1}^{Nh} \ln(sl_{ij}) - \mu \sum_{i=1}^{Nc} \sum_{j=1}^{Nh} \ln(su_{ij}) \\ & - \lambda \mathbf{g}_0^T \mathbf{g}_0(\mathbf{y}_0) - \boldsymbol{\pi} \mathbf{l}_0^T (\mathbf{h}_0(\mathbf{y}_0) - \mathbf{sl}_0 - \mathbf{h}_0^{\min}) \\ & - \boldsymbol{\pi} \mathbf{u}_0^T (\mathbf{h}_0(\mathbf{y}_0) + \mathbf{su}_0 - \mathbf{h}_0^{\max}) \\ & - \sum_i^{Nc} \lambda \mathbf{g}_i^T \mathbf{g}_i(\mathbf{y}_i) - \sum_i^{Nc} \boldsymbol{\pi} \mathbf{l}_i^T (\mathbf{h}_i(\mathbf{y}_i) - \mathbf{sl}_i - \mathbf{h}_i^{\min}) \\ & - \sum_i^{Nc} \boldsymbol{\pi} \mathbf{u}_i^T (\mathbf{h}_i(\mathbf{y}_i) + \mathbf{su}_i - \mathbf{h}_i^{\max}) \\ & - \sum_i^{Nc} \lambda \mathbf{u}_i^T (\mathbf{u}_0 - \mathbf{u}_i) \end{aligned} \quad (6.48)$$

where $\mu > 0$, $\mathbf{sl}_0 > \mathbf{0}$, $\mathbf{su}_0 > \mathbf{0}$, $\mathbf{sl}_i > \mathbf{0}$ and $\mathbf{su}_i > \mathbf{0}$. $\lambda \mathbf{g}_0$, $\pi \mathbf{l}_0$ and $\pi \mathbf{u}_0$ are dual variable vectors to equalities (6.41), (6.42) and (6.43), respectively. $\lambda \mathbf{g}_i$, $\pi \mathbf{l}_i$ and $\pi \mathbf{u}_i$ are dual variable vectors to equalities (6.44), (6.45) and (6.46), respectively. $\lambda \mathbf{u}_i$ is dual variable vector to equalities (6.47), which represent the constraints of the preventive controls. In (6.48), transformer tap ratios are treated as continuous variables.

The Karush-Kuhn-Tucker (KKT) first order conditions for the Lagrangian function of (6.48) are:

$$\begin{aligned} \nabla_{\mathbf{y}_0} L_\mu = & -\nabla_{\mathbf{y}_0} f(\lambda) - \nabla_{\mathbf{y}_0} \mathbf{g}_0(\mathbf{y}_0)^T \lambda \mathbf{g}_0 \\ & - \nabla_{\mathbf{y}_0} \mathbf{h}_0(\mathbf{y}_0)^T \pi \mathbf{l}_0 - \nabla_{\mathbf{y}_0} \mathbf{h}_0(\mathbf{y}_0)^T \pi \mathbf{u}_0 \\ & - \sum_i^{Nc} \nabla_{\mathbf{y}_0} \mathbf{u}_0^T \lambda \mathbf{u}_i \end{aligned} \quad (6.49)$$

$$\nabla_{\lambda \mathbf{g}_0} L_\mu = -\mathbf{g}_0(\mathbf{y}_0) = \mathbf{0} \quad (6.50)$$

$$\nabla_{\pi \mathbf{l}_0} L_\mu = -(\mathbf{h}_0(\mathbf{y}_0) - \mathbf{sl}_0 - \mathbf{h}_0^{\min}) \quad (6.51)$$

$$\nabla_{\pi \mathbf{u}_0} L_\mu = -(\mathbf{h}_0(\mathbf{y}_0) + \mathbf{su}_0 - \mathbf{h}_0^{\max}) \quad (6.52)$$

$$\nabla_{\mathbf{sl}_0} L_\mu = \mu \mathbf{e} - \mathbf{SL}_0 \Pi \mathbf{L}_0 \quad (6.53)$$

$$\nabla_{\mathbf{su}_0} L_\mu = \mu \mathbf{e} + \mathbf{SU}_0 \Pi \mathbf{U}_0 \quad (6.54)$$

$$\begin{aligned} \nabla_{\mathbf{y}_i} L_\mu = & -\nabla_{\mathbf{y}_i} f(\lambda) - \nabla_{\mathbf{y}_i} \mathbf{g}_i(\mathbf{y}_i)^T \lambda \mathbf{g}_i \\ & - \nabla_{\mathbf{y}_i} \mathbf{h}_i(\mathbf{y}_i)^T \pi \mathbf{l}_i - \nabla_{\mathbf{y}_i} \mathbf{h}_i(\mathbf{y}_i)^T \pi \mathbf{u}_i \\ & + \sum_i^{Nc} \nabla_{\mathbf{y}_i} \mathbf{u}_i^T \lambda \mathbf{u}_i \end{aligned} \quad (6.55)$$

$$\nabla_{\lambda \mathbf{g}_i} L_\mu = -\mathbf{g}_i(\mathbf{y}_i) = \mathbf{0} \quad (6.56)$$

$$\nabla_{\pi \mathbf{l}_i} L_\mu = -(\mathbf{h}_i(\mathbf{y}_i) - \mathbf{sl}_i - \mathbf{h}_i^{\min}) \quad (6.57)$$

$$\nabla_{\pi \mathbf{u}_i} L_\mu = -(\mathbf{h}_i(\mathbf{y}_i) + \mathbf{su}_i - \mathbf{h}_i^{\max}) \quad (6.58)$$

$$\nabla_{\mathbf{sl}_i} L_\mu = \mu \mathbf{e} - \mathbf{SL}_i \Pi \mathbf{L}_i \quad (6.59)$$

$$\nabla_{\mathbf{su}_i} L_\mu = \mu \mathbf{e} + \mathbf{SU}_i \Pi \mathbf{U}_i \quad (6.60)$$

$$\nabla_{\lambda \mathbf{u}_i} L_\mu = -(\mathbf{u}_0 - \mathbf{u}_i) \quad (6.61)$$

$$\nabla_{\lambda} L_\mu = -\frac{\partial f(\lambda)}{\partial \lambda} - \nabla_{\lambda} \mathbf{g}_0(\mathbf{y}_0)^T \mathbf{e} - \nabla_{\lambda} \mathbf{g}_i(\mathbf{y}_i)^T \mathbf{e} \quad (6.62)$$

where $i = 1, 2, \dots, N_c$. $\mathbf{SL}_0 = \text{diag}(sl_{0j})$, $\mathbf{SU}_0 = \text{diag}(su_{0j})$, $\mathbf{\Pi L}_0 = \text{diag}(\pi l_{0j})$, $\mathbf{\Pi U}_0 = \text{diag}(\pi u_{0j})$, $\mathbf{SL}_i = \text{diag}(sl_{ij})$, $\mathbf{SU}_i = \text{diag}(su_{ij})$, $\mathbf{\Pi L}_i = \text{diag}(\pi l_{ij})$, $\mathbf{\Pi U}_i = \text{diag}(\pi u_{ij})$.

The nonlinear equations (6.49)-(6.62) in polar coordinates can be solved simultaneously. The simultaneous equations can be linearized and expressed in a compact Newton form:

$$\mathbf{A} \Delta \mathbf{x} = -\mathbf{b} \quad (6.63)$$

where $\mathbf{A} = \frac{\partial \mathbf{b}}{\partial \mathbf{x}}$. $\mathbf{x} = [\mathbf{x}_0, \mathbf{x}_i, \lambda \mathbf{u}_i, \lambda]^T$. $\mathbf{b} = [\mathbf{b}_0, \mathbf{b}_i, \mathbf{b} \mathbf{u}_i, b_\lambda]^T$. $\mathbf{X}_0$ and $\mathbf{X}_i$ are given by:

$$\mathbf{x}_0 = [\mathbf{sl}_0, \mathbf{su}_0, \mathbf{\pi l}_0, \mathbf{\pi u}_0, \mathbf{y}_0, \lambda \mathbf{g}_0]^T \quad (6.64)$$

$$\mathbf{x}_i = [\mathbf{sl}_i, \mathbf{su}_i, \mathbf{\pi l}_i, \mathbf{\pi u}_i, \mathbf{y}_i, \lambda \mathbf{g}_i]^T \quad (6.65)$$

and $\mathbf{b}_0$, $\mathbf{b}_i$, $\mathbf{b} \mathbf{u}_i$ and b_λ are given by:

$$\mathbf{b}_0 = [\nabla_{\mathbf{sl}_0} L_\mu, \nabla_{\mathbf{su}_0} L_\mu, \nabla_{\mathbf{\pi l}_0} L_\mu, \nabla_{\mathbf{\pi u}_0} L_\mu, \nabla_{\mathbf{y}_0} L_\mu, \nabla_{\lambda \mathbf{g}_0} L_\mu]^T \quad (6.66)$$

$$\mathbf{b}_i = [\nabla_{\mathbf{sl}_i} L_\mu, \nabla_{\mathbf{su}_i} L_\mu, \nabla_{\mathbf{\pi l}_i} L_\mu, \nabla_{\mathbf{\pi u}_i} L_\mu, \nabla_{\mathbf{y}_i} L_\mu, \nabla_{\lambda \mathbf{g}_i} L_\mu]^T \quad (6.67)$$

$$\mathbf{b} \mathbf{u}_i = \nabla_{\lambda \mathbf{u}_i} L_\mu \quad (6.68)$$

$$b_\lambda = \nabla_{\lambda} L_\mu \quad (6.69)$$

The security constrained TC problem can be solved iteratively via the Newton equation in (6.63), and at each iteration the solution can be updated as follows:

$$\mathbf{sl}_0[k+1] = \mathbf{sl}_0[k] + \sigma \alpha_p \Delta \mathbf{sl}_0[k] \quad (6.70)$$

$$\mathbf{su}_0[k+1] = \mathbf{su}_0[k] + \sigma \alpha_p \Delta \mathbf{su}_0[k] \quad (6.71)$$

$$\mathbf{y}_0[k+1] = \mathbf{y}_0[k] + \sigma \alpha_p \Delta \mathbf{y}_0[k] \quad (6.72)$$

$$\mathbf{\pi l}_0[k+1] = \mathbf{\pi l}_0[k] + \sigma \alpha_d \Delta \mathbf{\pi l}_0[k] \quad (6.73)$$

$$\mathbf{\pi u}_0[k+1] = \mathbf{\pi u}_0[k] + \sigma \alpha_d \Delta \mathbf{\pi u}_0[k] \quad (6.74)$$

$$\mathbf{sl}_i[k+1] = \mathbf{sl}_i[k] + \sigma \alpha_p \Delta \mathbf{sl}_i[k] \quad (6.75)$$

$$\mathbf{su}_i[k+1] = \mathbf{su}_i[k] + \sigma\alpha_p \Delta \mathbf{su}_i[k] \quad (6.76)$$

$$\mathbf{y}_i[k+1] = \mathbf{y}_i[k] + \sigma\alpha_p \Delta \mathbf{y}_i[k] \quad (6.77)$$

$$\boldsymbol{\pi l}_i[k+1] = \boldsymbol{\pi l}_i[k] + \sigma\alpha_d \Delta \boldsymbol{\pi l}_i[k] \quad (6.78)$$

$$\boldsymbol{\pi u}_i[k+1] = \boldsymbol{\pi u}_i[k] + \sigma\alpha_d \Delta \boldsymbol{\pi u}_i[k] \quad (6.79)$$

$$\boldsymbol{\lambda u}_i[k+1] = \boldsymbol{\lambda u}_i[k] + \sigma\alpha_d \Delta \boldsymbol{\lambda u}_i[k] \quad (6.80)$$

$$\lambda[k+1] = \lambda[k] + \sigma\alpha_p \Delta \lambda[k] \quad (6.81)$$

where $i = 1, 2, \dots, Nc$. k is the iteration count. Parameter $\sigma \in [0.995-0.99995]$. α_p and α_d are the primal and dual step-length parameters, respectively. The step-lengths are determined as follows:

$$\alpha p_0 = \min \left[\min \left(\frac{sl_{0j}}{-\Delta sl_{0j}} \right), \min \left(\frac{su_{0j}}{-\Delta su_{0j}} \right), 1.00 \right] \quad (6.82)$$

$$\alpha d_0 = \min \left[\min \left(\frac{\pi l_{0j}}{-\Delta \pi l_{0j}} \right), \min \left(\frac{\pi u_{0j}}{-\Delta \pi u_{0j}} \right), 1.00 \right] \quad (6.83)$$

$$\alpha p_i = \min \left[\min \left(\frac{sl_{ij}}{-\Delta sl_{ij}} \right), \min \left(\frac{su_{ij}}{-\Delta su_{ij}} \right), 1.00 \right] \quad (6.84)$$

$$\alpha d_i = \min \left[\min \left(\frac{\pi l_{ij}}{-\Delta \pi l_{ij}} \right), \min \left(\frac{\pi u_{ij}}{-\Delta \pi u_{ij}} \right), 1.00 \right] \quad (6.85)$$

$$i = 1, 2, \dots, Nc$$

for those $\Delta sl < 0$, $\Delta su < 0$, $\Delta \pi l < 0$ and $\Delta \pi u > 0$. α_p and α_d are determined by:

$$\alpha_p = \min[\alpha p_0, \alpha p_i] \quad (i = 1, 2, \dots, Nc) \quad (6.86)$$

$$\alpha_d = \min[\alpha d_0, \alpha d_i] \quad (i = 1, 2, \dots, Nc) \quad (6.87)$$

The Barrier parameter μ can be evaluated by:

$$\mu = \frac{\beta \times Cgap}{2 \times Nh \times (Nc + 1)} \quad (6.88)$$

where $\beta \in [0.01-0.2]$ and $Cgap$ is the complementary gap for the transfer capability calculation problem with security constraints. It can be determined by:

$$Cgap = (\mathbf{sl}_0)^T \boldsymbol{\pi l}_0 - (\mathbf{su}_0)^T \boldsymbol{\pi u}_0 + \sum_{i=1}^{N_c} [(\mathbf{sl}_i)^T \boldsymbol{\pi l}_i - (\mathbf{su}_i)^T \boldsymbol{\pi u}_i] \quad (6.89)$$

6.3.3 Solution Procedure of the Security Constrained Transfer Capability Problem

The solution procedure of the nonlinear interior point optimization algorithm for the unified security constrained transfer capability problem is summarized as follows:

- Step 0:* Set iteration count $k = 0$, $\mu = \mu_0$, and initialize the optimization solution
- Step 1:* If KKT conditions (6.49)–(6.62) are satisfied and the complementary gap is less than a tolerance, output results. Otherwise go to step 2
- Step 2:* Form and solve Newton equation in (6.63)
- Step 3:* Update Newton solution (6.70)–(6.81)
- Step 4:* Compute complementary gap (6.89)
- Step 5:* Determine barrier parameter (6.88)
- Step 6:* Set $k=k+1$, and go to step 1

6.3.4 Numerical Results

Test cases are carried out on the IEEE 30-bus system. The IEEE 30-bus system has 6 generators, 4 OLTC transformers and 37 transmission lines. The single-line diagram of the IEEE 30-bus system is shown in Fig. 2.2. For all cases tested, the convergence criteria are:

1. Complementary gap $Cgap \leq 5.0e^{-4}$
2. Barrier parameter $\mu \leq 1.0e^{-4}$
3. Maximum mismatch of the Newton equation $\|\mathbf{b}\|_{\infty} \leq 1.0e^{-4}$ p.u.

6.3.4.1 IEEE 30-Bus System Results

In the simulations, it is assumed that all generators except the generator at bus 1 are using preventive control of active power generation while other control resources are using corrective controls. Bus 1 is the slack bus.

In the study, the IEEE 30-bus system was divided into two areas. The two areas are interconnected by tie-lines 4-12, 6-9, 6-10, and 28-27 while buses 4, 6, 28 be-

long to the area 1. The power transfer from the area 1 to the area 2 has been investigated. The single state cases are presented as follows:

Case 1: This is a base case for the transfer capability computation.

Case 2: This is similar to Case 1 except that there is an outage of line 5-7.

Case 3: This is similar to Case 1 except that there is an outage of line 24-25.

The transfer capabilities of Case 1-3 on the IEEE 30 bus system are shown in Table 6.5.

Table 6.5. Transfer Capability Results of Single State Cases

Case No.			Case 1	Case 2	Case 3
Transmission description	line	outage	None	Line 5-7	Line 24-25
λ			1.64	1.49	1.63
Number of iterations			14	15	16

The transfer capability results of cases with security constraints on the IEEE 30-bus system are presented as follows:

Case 4: This is a case for the transfer capability computation including one $N-1$ contingency with line 5-7 outage.

Case 5: This is a case for the transfer capability computation including one $N-1$ contingency with line 24-25 outage.

Case 6: This is a case for the transfer capability computation with two contingencies. The first contingency is the outage of line 5-7 while the second one is the outage of line 24-25.

The transfer capabilities of Case 4-6 are shown in Table 6.6. From this Table, it can be seen the CPU time for the transfer capability calculations with security constraints is proportional to the total number of base case and contingencies.

Cases 7 and 8 are presented to show the security constrained transfer capability computation with FACTS. Cases 7 and 8 are corresponding to Cases 4 and 6, respectively except that an UPFC is installed between buses 3 and 4. The test results of Cases 7 and 8 are shown in Table 6.7. The UPFC solutions of Cases 7 and 8 are given by Table 6.8. In the calculations, the UPFC is using corrective controls. The results indicate that UPFC taking corrective control actions can improve the transfer capability effectively.

A further case, case 9, is carried out with 8 contingencies included in the transfer capability computation. The algorithm converges in 15 iterations. The $N-1$ contingencies are outages of lines 2-6, 4-6, 5-7, 6-7, 10-21, 12-15, 12-16 and 24-25, respectively. It is found for this case $\lambda=1.05$.

It can also be seen that in the above cases, the more contingencies, the less system transfer capability is available.

6.3.4.2 Discussion of the Results

From these results on the IEEE 30-bus system, it can be seen:

1. Numerical results demonstrate the feasibility of the proposed unified optimization framework for transfer capability computation with security constraints.
2. The computation framework is general, which can simultaneously take voltage, thermal and voltage stability limits as well as any electricity transaction constraints into consideration.
3. The optimization framework of transfer capability computation with security constraints can be solved by nonlinear interior point methods.
4. FACTS-devices can be modeled as corrective control devices in the calculations.
5. In addition, electricity transaction constraints may be taken into consideration.

Table 6.6. Transfer Capability Results of Cases with Security Constraints

Case No.	Case 4	Case 5	Case 6
Transmission line outage description	Base case and one N-1 contingency with line 5-7 outage	Base case and one N-1 contingency with line 24-25 outage	Base case and two N-1 contingencies: with line 24-25 outage and line 5-7 outage
λ	1.49	1.63	1.49
Number of iterations	14	14	15
Normalised CPU time	100%	100%	150%

Table 6.7. Transfer Capability Results of Cases with FACTS

Case No.	Case 7	Case 8
Transmission line outage description	Base case and one N-1 contingency with line 5-7 outage	Base case and two N-1 contingencies with line 24-25 outage and line 5-7 outage, respectively
λ	1.75	1.64
Number of iterations	28	32

Table 6.8. UPFC solutions of Cases 7 and 8

Case 7	Case 8
Base case:	Base case:
Shunt converter: $V_{sh} = 0.9786 \text{ p.u.}$, $\theta_{sh} = -10.61^\circ$	Shunt converter: $V_{sh} = 0.9596 \text{ p.u.}$, $\theta_{sh} = -7.54^\circ$
Series converters: $V_{se} = 0.1829 \text{ p.u.}$, $\theta_{se} = 118.05^\circ$	Series converters: $V_{se} = 0.1033 \text{ p.u.}$, $\theta_{se} = 179.54^\circ$
Contingency with line 5-7 outage:	Contingency with line 5-7 outage:
Shunt converter: $V_{sh} = 0.9707 \text{ p.u.}$, $\theta_{sh} = -12.86^\circ$	Shunt converter: $V_{sh} = 0.9615 \text{ p.u.}$, $\theta_{sh} = -7.51^\circ$
Series converters: $V_{se} = 0.2503 \text{ p.u.}$, $\theta_{se} = -114.30^\circ$	Series converters: $V_{se} = 0.1006 \text{ p.u.}$, $\theta_{se} = -179.67^\circ$
	Contingency with line 24-25 outage:
	Shunt converter: $V_{sh} = 0.9685 \text{ p.u.}$, $\theta_{sh} = -9.85^\circ$
	Series converters: $V_{se} = 0.1280 \text{ p.u.}$, $\theta_{se} = -145.05^\circ$

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